Neighbourhood interaction in spatio-temporal models

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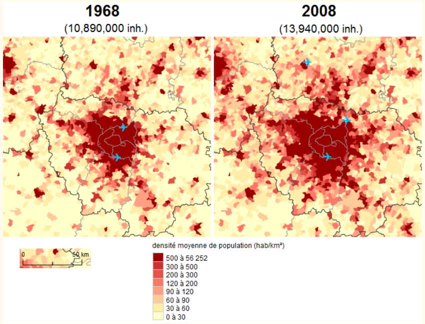
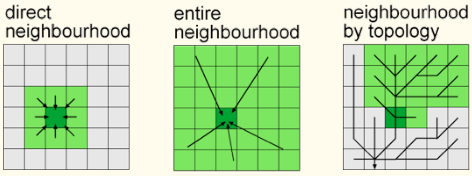
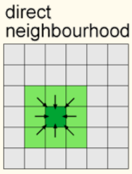


Neighbourhood relations



Direct neighbourhood operations: cellular automata



Principle of cellular automata

Local (spatial) interactions create global pattern of change

Example: vegetation patterns

local

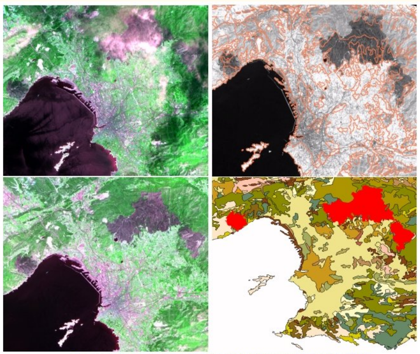
global

Cellular automata: forest fire

Local processes

Cellular automata: forest fire

Global effect (Marseille, France)



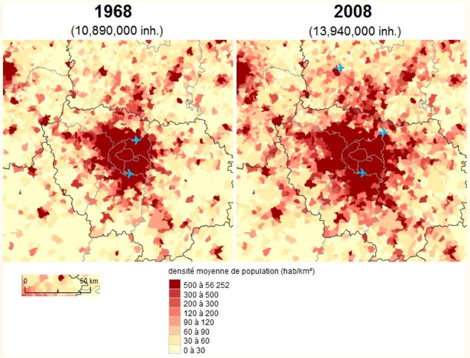
Burnt area mapping and forest fire damage assessment (Marseille, 1997)

Cellular automata: urban sprawl

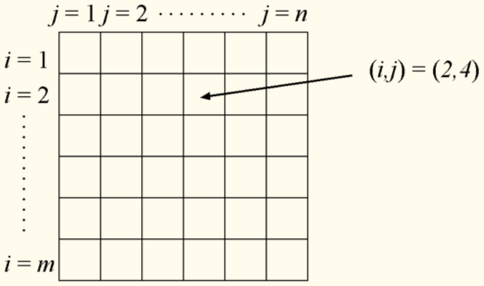
Local processes (Leidse Rijn, Utrecht)

Cellular automata: urban sprawl

Global effect (Paris)



Spatial discretization: regular lattice of cells



Formulating cellular automata

Cellular automata:

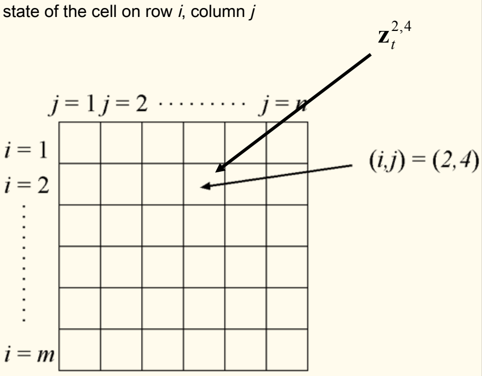
$$\mathbf{z}_{t+1} = \mathbf{f}(\mathbf{z}_t) \quad \text{for each } t$$

- No inputs (although in practice inputs may be used)
- All variables in **z** are classified (although scalars are sometimes used)
- **f** is a function (‘transition rule’) of a direct neighbourhood and the cell itself only
- The change from *t* to *t* + 1 is not a function of time

What is **f**?

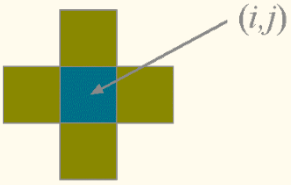
Notation rules: referring to neighbouring cells

$\mathbf{z}_t^{ij}$ the state of the cell on row *i*, column *j*



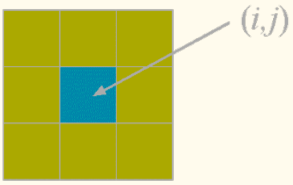
Transition rule: von Neumann Neighborhood

$$\mathbf{z}_{t+1}^{ij} = \mathbf{f}\left(\mathbf{z}_t^{i-1,j}, \mathbf{z}_t^{i,j+1}, \mathbf{z}_t^{i+1,j}, \mathbf{z}_t^{i,j-1}, \mathbf{z}_t^{i,j}\right) \quad \text{for each } t$$



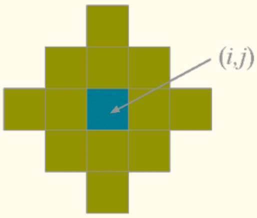
Transition rule: Moore neighborhood

$$\mathbf{z}_{t+1}^{ij} = \mathbf{f}\big(\mathbf{z}_t^{i-1,j-1}, \mathbf{z}_t^{i-1,j}, \mathbf{z}_t^{i-1,j+1}, \mathbf{z}_t^{i,j+1}, \mathbf{z}_t^{i+1,j+1}, \mathbf{z}_t^{i+1,j}, \mathbf{z}_t^{i+1,j-1}, \mathbf{z}_t^{i,j-1}, \mathbf{z}_t^{i,j}\big) \quad \text{for each } t$$



Transition rule: extended von Neumann neighborhood

$$\mathbf{z}_{t+1}^{ij} = \mathbf{f}\big(\mathbf{z}_t^{i-1,j-1}, \mathbf{z}_t^{i-1,j}, \mathbf{z}_t^{i-1,j+1}, \mathbf{z}_t^{i,j+1}, \mathbf{z}_t^{i+1,j+1}, \mathbf{z}_t^{i+1,j}, \mathbf{z}_t^{i+1,j-1}, \mathbf{z}_t^{i,j-1}, \mathbf{z}_t^{i-2,j}, \mathbf{z}_t^{i,j+2}, \mathbf{z}_t^{i+2,j}, \mathbf{z}_t^{i,j-2}, \mathbf{z}_t^{i,j}\big) \quad \text{for each } t$$

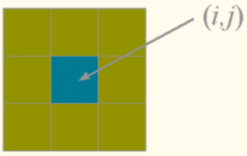


Life, also known as Game of Life

Developed by John Conway, 1970

One boolean variable: TRUE (= live cell) or FALSE (= dead cell)

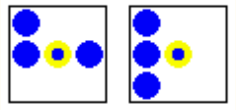
Uses the Moore neighborhood



Life, also known as Game of Life

Transition rules (i.e. **f**)

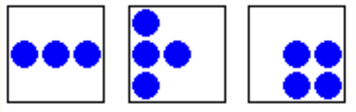
- a dead cell with exactly three alive neighbors becomes a live cell (birth)



Life, also known as Game of Life

Transition rules (i.e. **f**)

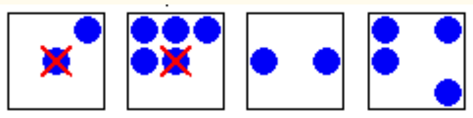
- a dead cell with exactly three alive neighbors becomes a live cell (birth)
- a live cell with two or three neighbors stays alive (survival)



Life, also known as Game of Life

Transition rules (i.e. **f**)

- a dead cell with exactly three alive neighbors becomes a live cell (birth)
- a live cell with two or three neighbors stays alive (survival)
- in all other cases, a cell dies or remains dead



LIFE in PCRaster

```
initial
 Alive=uniform(1) gt 0.5;

dynamic
 NumberOfAliveNeighbors=windowtotal(scalar(Alive),3)-
                                                  scalar(Alive);
 Alive=(NumberOfAliveNeighbors eq 3) or
                 ((NumberOfAliveNeighbors eq 2) and Alive);
```

Demo

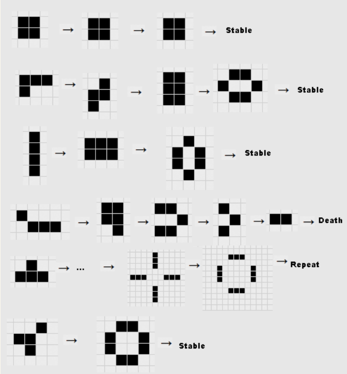
game of life

edit tot.mod

display ini.map

aguila –2 alive000.001+1000 test0000.001+1000

Stable, repeating, and dying patterns



From: http://www.math.cornell.edu/~lipa/mec/lesson6.html

Self-organization

Internal organization of a system increases in complexity without being guided by an input variable

Emergent properties

Example: game of life

Many examples from ecology, sedimentology, land degradation

Other neighbourhood relations