Calibration of Environmental Models

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Calibration e-lectures

1. Introduction & manual calibration
2. Automatic calibration (1): objective function & response surfaces
3. Automatic calibration (2): calibration algorithms

Calibration

- Introduction
- Manual calibration

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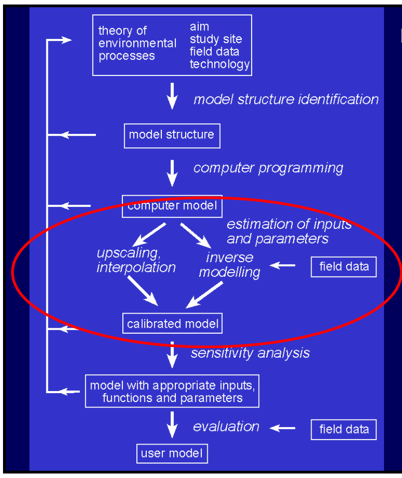
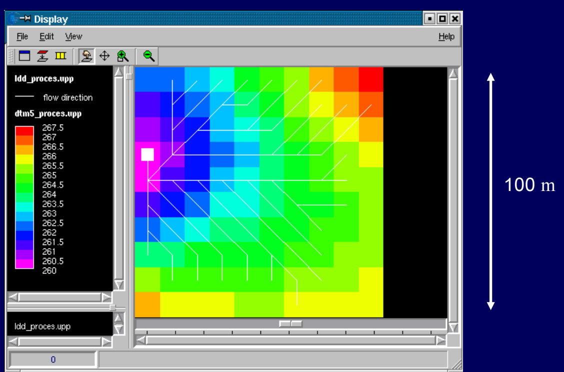
Model development cycle



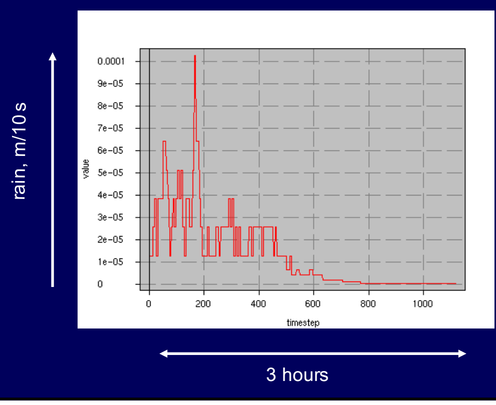
Example model: rainfall-runoff model of a hillslope (France)

- Digital Elevation Model



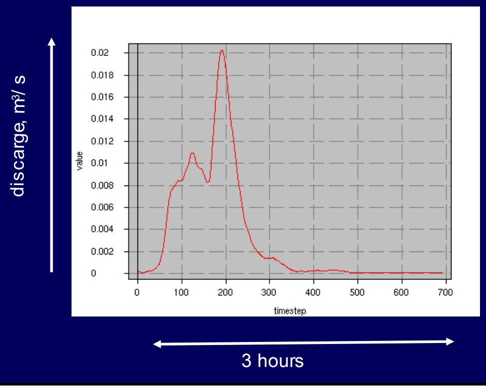
Rainfall-runoff model of a hillslope (France)

- Observed rainfall



Rainfall-runoff model of a hillslope (France)

- Observed discharge at outflow point



Model structure

- Rainfall (timeseries)
- Infiltration
  constant infiltration capacity
  parameter: K_{Sat} (mm/h)
- Runoff
  Manning equation (kinematic wave)
  parameter: n

timestep: 10 seconds, cellsize 10 m

```
dynamic
 # rain per timestep (m/timestep)
 Pr=timeinputscalar(RainTSS,Clone)

 # flow out off the cell (m/timestep)
 QR=(Q*T)/CA
 # flow into the cell, from non channel cells (m/timestep)
 QRNCh=upstream(Ldd,QR)
 SurW=Pr+QRNCh

 # infiltration
 SurW=SurW-I;

 # lateral inflow (m3/s)
 QIn=((SurW-QRNCh)*CA)/T;
 # per distance along stream ((m3/s)/m))
 q=QIn/DCL;
 Q = max(0.0001,Q);

 ...
```

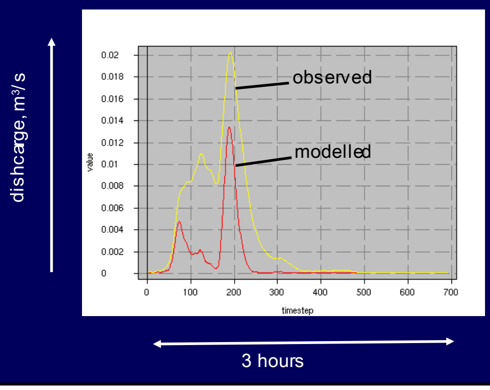
```
...

 # discharge (m3/s)
 Q=kinematic(Ldd,Q,q,Alpha,Beta,T,DCL)

 # water depth (m)
 H=(Alpha*(Q**Beta))/Bw
 # wetted perimeter (m)
 P=Bw+2*H
 # Alpha
 Alpha=AlpTerm*(P**AlpPow)
```

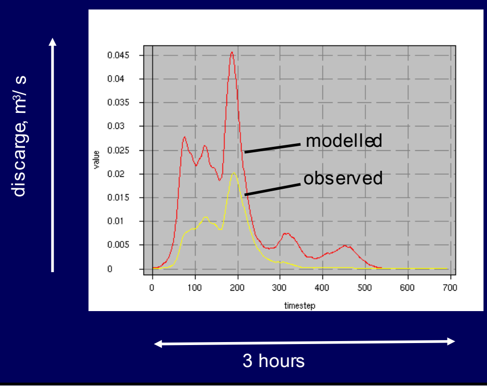
Model run with measured (*KSat*) and tabulated value (*n*)

- K_{sat} = 30 mm/h, n = 0.038



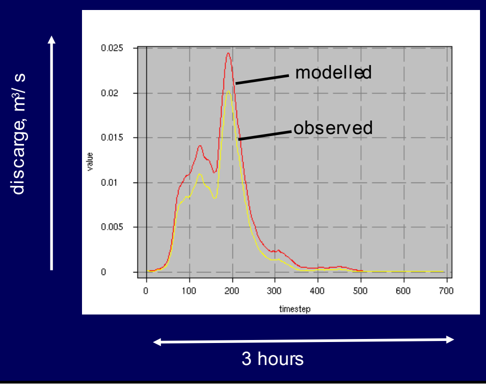
Model run with measured (*KSat*) and tabulated value (*n*)

- adjusted K_{sat} = 15 mm/h, n = 0.038



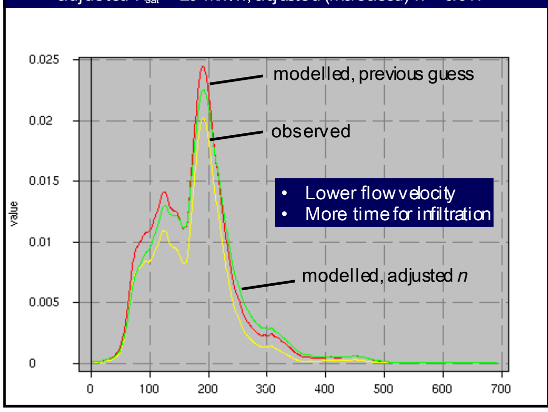
Model run with measured (*KSat*) and tabulated value (*n*)

- adjusted K_{sat} = 20 mm/h, n = 0.038



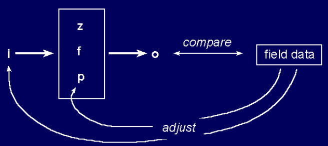
Model run with measured (*KSat*) and tabulated value (*n*)

- adjusted K_{sat} = 20 mm/h, adjusted (increased) n = 0.047



Calibration

Finding inputs or parameters by minimizing the difference between model outputs and measurements of these outputs



z state variables
i inputs
f functionals
p parameters
o outputs
i.e. a set of state variables in which the interest lies

Calibration, manual adjustment of parameters

Approach

- Visual comparison between observed and modelled outputs
- Manual adjustment of parameters (trial and error) to minimize difference between observed and modelled outputs

Disadvantages:

- Subjective
- Takes a lot of time
- It is difficult to find the 'best' values, particularly with multiple parameters
- No information on the uncertainty of the estimated parameters

Calibration

- Automatic calibration (1)
  - Objective function
  - Response surface

Calibration, automatic adjustment

Approach:

- Define an objective function (also, goal function)
- Calibrate the parameters resulting in the lowest (highest) value of the goal function
- Calibration is done with a computer algorithm

Objective function

Provides a quantitative measure of the goodness of fit between (the) model output(s) and observed values of the corresponding variables

Example: mean square error (MSE)

$$MSE = \frac{\sum_{t=1}^{n}\left(\hat{z}_t - z_t\right)^2}{n}$$

$\hat{z}_t$ modelled variable at t
z_t measured variable at t
n number of timesteps

Objective function

Other examples:

$$E_a = |\hat{q}_s - q_s|$$

E_a: absolute error in total runoff; $\hat{q}_s$: modelled total runoff; q_s: observed total runoff

$$E_{tt} = |\widehat{q_{tt}} - q_{tt}|$$

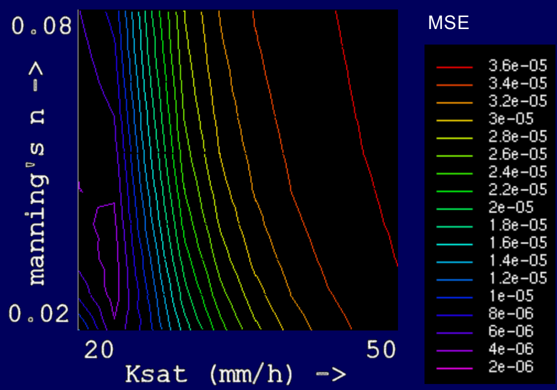
E_{tt}: absolute error in time to peak; $\widehat{q_{tt}}$: modelled time to peak; q_{tt}: observed time to peak

MSE in our example

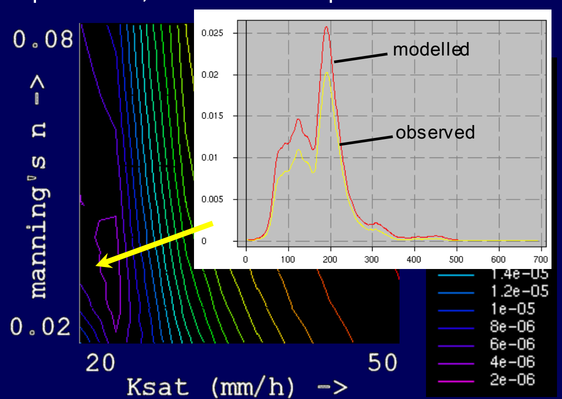
$$MSE = \frac{\sum_{t=1}^{n}\left(\hat{z}_t - z_t\right)^2}{n}$$

$\hat{z}_t$ modelled discharge (m^3/s) at t
z_t measured discharge (m^3/s) at t
n number of timesteps, 691 (each time step is 5 s)

Response surface, MSE value in the example

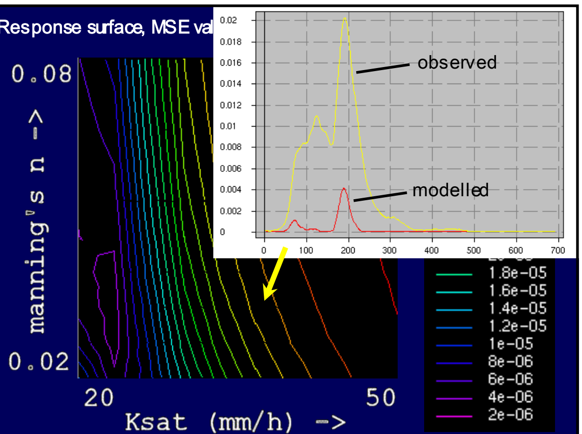


Response surface, MSE value in the example

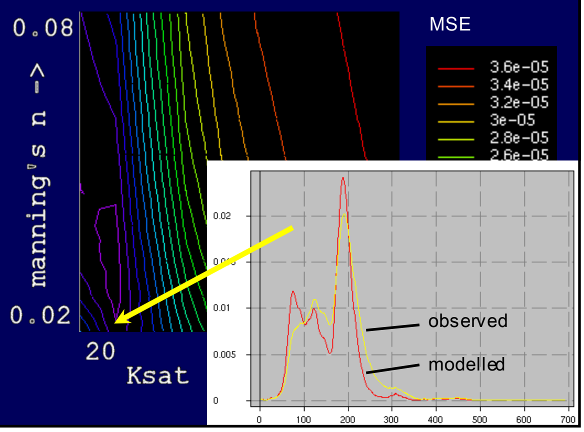


goal function

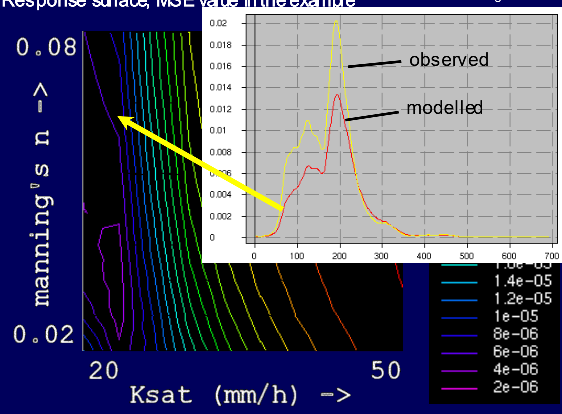
Response surface, MSE va



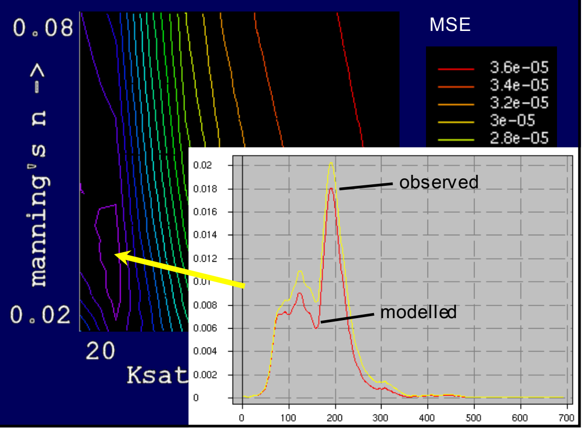
Response surface, MSE value in the example



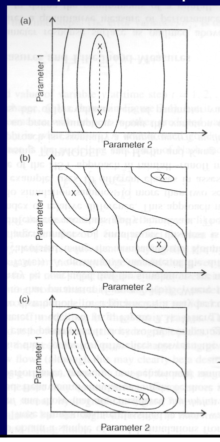
Response surface, MSE value in the example



Response surface, MSE value in the example



Examples of other shapes of a 2D response surfaces



Values in objective function do not (or hardly) depend on parameter 1

- Example: cloudiness factor in an event based rainfall-runoff model

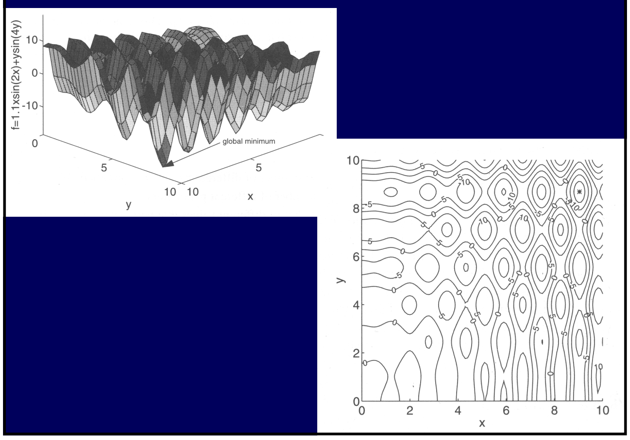
Multiple combinations of parameter values all give good objective function values

- Example: models with many parameters

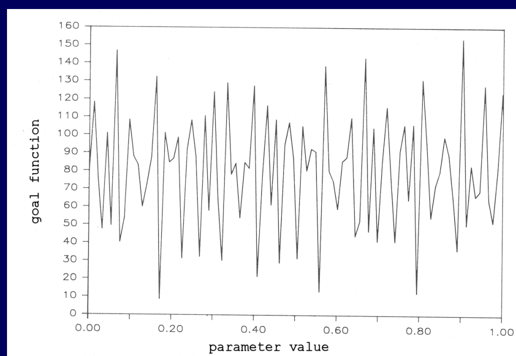
Increasing Parameter 1 has similar effect as decreasing Parameter 2

- Example K_{sat} and Manning's *n*

Examples of other shapes of a 2D response surfaces



1D respons surface with many local minima



Higher-dimensional response surfaces

goal function

When several parameters are unknown, e.g.

- saturated conductivity of several soil layers
- vegetation cover of several vegetation units
- maximum interception store
- surface storage of several soil units
- manning's *n*
- groundwater flow parameters
- etc..

Calibration

- Automatic calibration (2)
  - Calibration algorithms
- Wrap-up

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Calibration, automatic adjustment

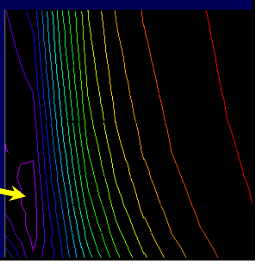
automatic adjustment

Approach:

- Define a goal function
- Optimize the parameters resulting in the lowest (highest) value of the goal function

i.e.:
how do we find the set of parameter values resulting in the lowest (highest) value of the goal function

or, in other words:
how do we find the minimum (or maximum) of the response surface



Calibration, automatic adjustment

automatic adjustment

Approach:

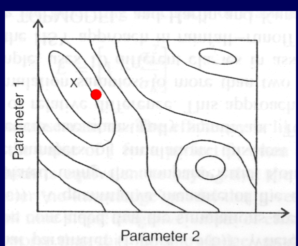
- Define a goal function
- Optimize the parameters resulting in the lowest (highest) value of the goal function
- Optimization is done with a computer algorithm
  - brute force
  - hill-climbing techniques
  - genetic algorithms

Choice of optimization algorithms

automatic adjustment

Important is:

- How close does the algorithm get to the real minimum value of the goal function (response surface)?



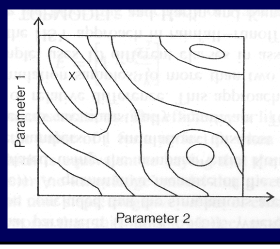
Choice of optimization algorithms

automatic adjustment

Important is:

- How close does the algorithm get to the real minimum value of the goal function (response surface)?

- Is the global minimum found or just a local minimum?



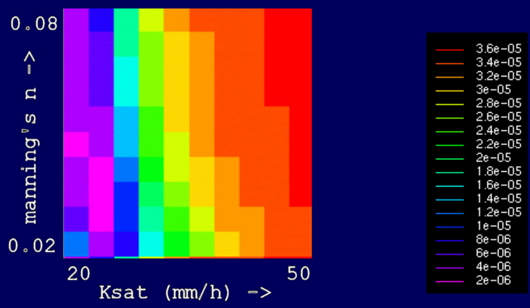
Choice of optimization algorithms

Important is:

- How close does the algorithm get to the real minimum value of the goal function (response surface)?
- Is the global minimum found or just a local minimum?
- How many model runs are needed to find the minimum?

Brute force approach

1. Run the model for a large set of parameter value combinations
2. Select the combination with the lowest value of the goal function



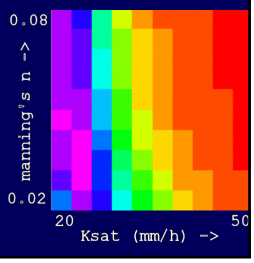
Brute force approach

Advantages:

- simple
- 'whole' respons surface is calculated
- (large) local minima are found

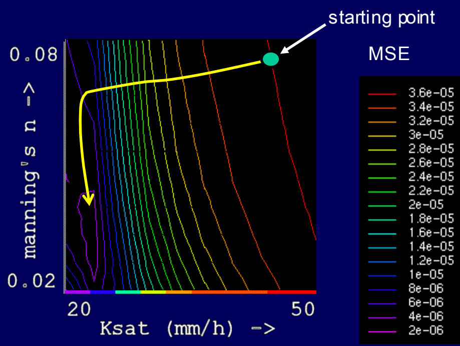
Disadvantages:

- Local minima are missed when small
- Optimization is not done inbetween the steps for parameter values used
- Many model runs are needed



Hill-climbing techniques

Use the shape of the response surface to reach the minimum value



Hill-climbing techniques

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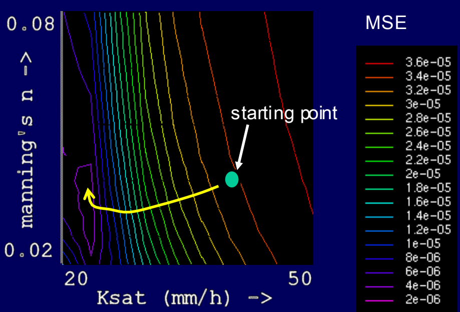
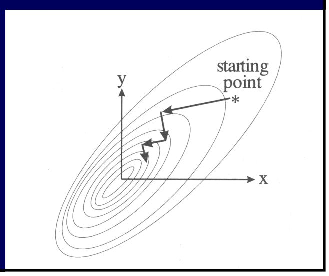


Hill-climbing techniques, steps in algorithm:

1. Choose for each parameter a starting value (= location on the response surface)
2. Calculate the gradient of the response surface at that location (by running the model with slightly different parameter values)
3. Go in the direction of this gradient over the response surface to a new location, if minimum is found, stop, or else continue at 2



Hill-climbing techniques

advantages:

- Small number of runs needed (compared to brute force)
- Location of minimum can be found with high precision

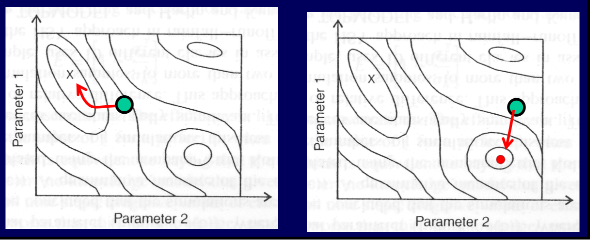
Hill-climbing techniques

Advantages:

- Small number of runs needed (compared to brute force)
- Location of minimum can be found with high precision

Disadvantages

- Danger exists that only a local minimum is found (search is always downhill



Genetic algorithms

Advantages:

- Capable to search in many local minima
- Relatively small number of model runs (compared to brute force)

Disadvantages

- Not possible (or very difficult) to describe the value of the outcome by means of statistics

Wrap-up - choice of the optimization algorithm

Simple problems:

- brute force
- hill climbing approach
    - standard software available (PEST)

Multiple local minima:

- genetic algorithm (not explained in this course)
- or combination of hill climbing and genetic algorithm