

SPATIO-TEMPORAL SIMULATION MODELLING: POINT MODELS AND DIFFERENTIAL EQUATIONS

02 Euler and Heun's methods

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Numerical solution
Many numerical solution algorithms are available

- Euler method
- Heun's method
- Runge-Kutta method

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Euler method or Euler-Cauchy method

The solution of the initial value problem

$$\frac{dy}{dt} = f(y, t), \quad y(t_0) = c$$

Is (Euler or Euler-Cauchy method):

$$y(t + \Delta t) = y(t) + \Delta t \cdot f(y(t), t)$$

with:

Δt time step length

Euler

Euler

Euler method or Euler-Cauchy method

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Is (Euler or Euler-Cauchy method):

$$y(t + \Delta t) = y(t) + \Delta t \cdot f(y(t), t)$$

with:

Δt time step length

Exercise:

$$\frac{dy}{dt} = ky, \quad y(t_0) = y_i$$

$$y(t_0) = 0.02$$

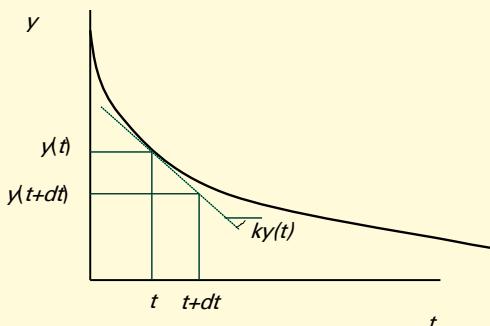
$$k = -0.002$$

Calculate $y(t + 60)$

Use a time step of 60

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Euler method or Euler-Cauchy method, example

We have the initial value problem

$$\frac{dy}{dt} = ky, \quad y(t_0) = y_i$$

The solution is: (Euler or Euler-Cauchy method):

$$y(t + \Delta t) = y(t) + \Delta t \cdot (ky)$$

Note: this is how we solved it initially

Euler

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Euler method or Euler-Cauchy method, example

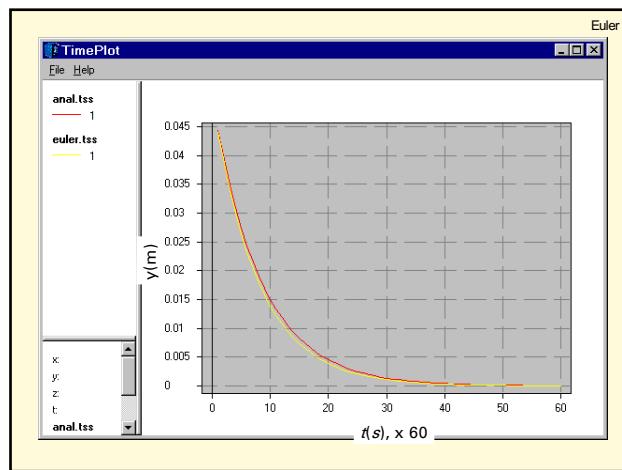
k = -0.002;           # fraction output from canopy (s-1)
Dt = 60;              # timestep (s)

y = 0.05
t = 0

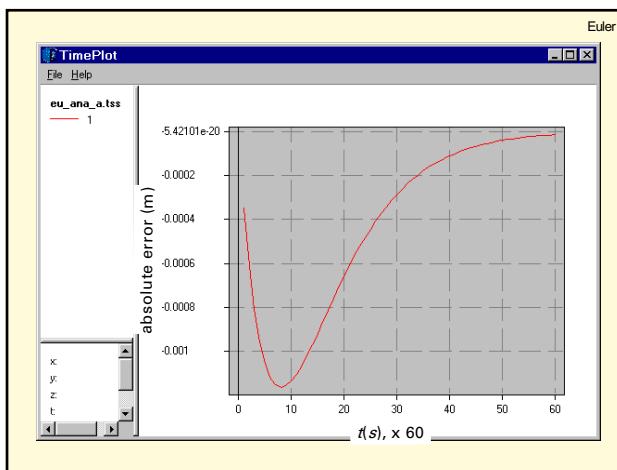
while t < 60
    y = y + Dt * k * y # y is on the right side!
    print(y)
    t = t + 1

```

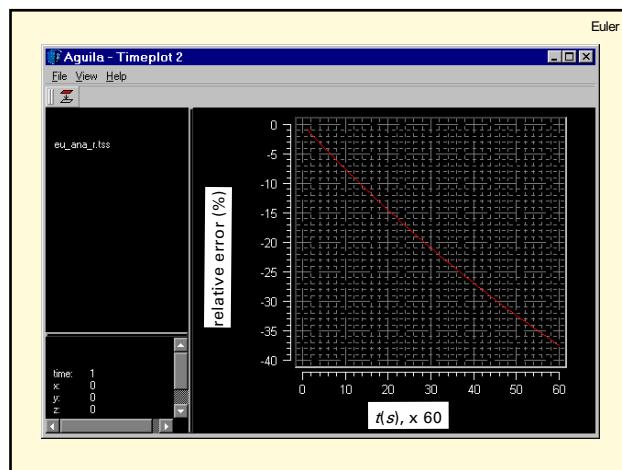
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Heun's method

The solution of the initial value problem

$$\frac{dy}{dt} = f(y, t), \quad y(t_0) = c$$

is:

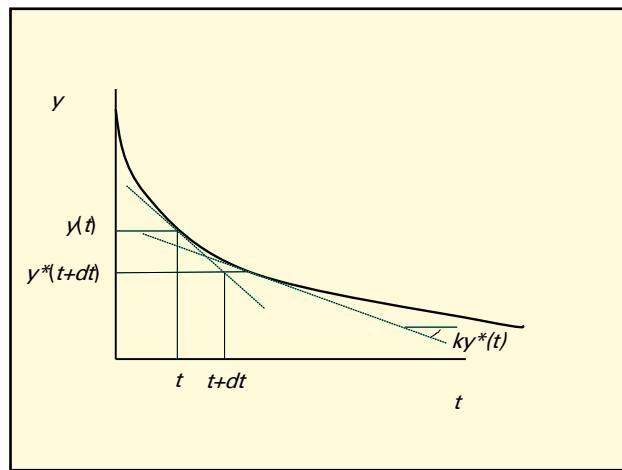
$$y(t + \Delta t) = y(t) + \Delta t \cdot \frac{f(y(t), t) + f(y^*(t + \Delta t), t + \Delta t)}{2},$$

with:

$$y^*(t + \Delta t) = y(t) + \Delta t \cdot f(y(t), t)$$

(note: $y^*(t + \Delta t)$ is calculated with Euler's method)

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Exercise

The solution of the initial value problem

$$\frac{dy}{dt} = f(y, t), \quad y(t_0) = c$$

is:

$$y(t + \Delta t) = y(t) + \Delta t \cdot \frac{f(y(t), t) + f(y^*(t + \Delta t), t + \Delta t)}{2},$$

with:

Exercise:
 $\frac{dy}{dt} = ky,$ $y(t_0) = y_i$
 $y(t_0) = 0.02$
 $k = -0.002$
Calculate $y(t + 60)$
Use a time step of 60

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Heun's method, example

We have the initial value problem

$$\frac{dy}{dt} = ky, \quad y(t_0) = y_i$$

The solution is:

$$y(t + \Delta t) = y(t) + \Delta t \cdot \frac{ky + ky^*}{2},$$

with

$$y^*(t + \Delta t) = y(t) + \Delta t \cdot (ky)$$

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Heun's method, example

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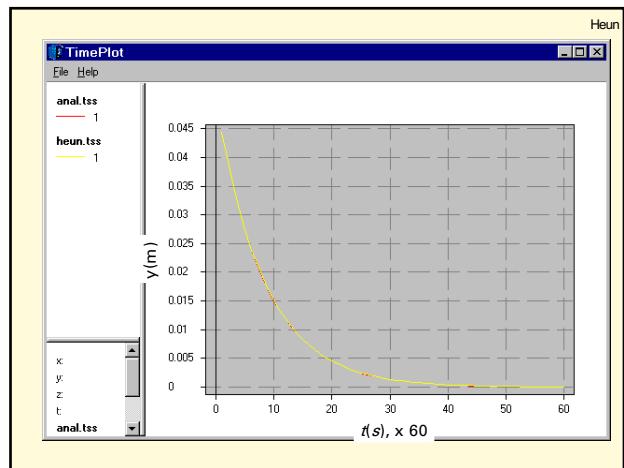
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Dt = 60;              # timestep (s)

y = 0.05
t = 0

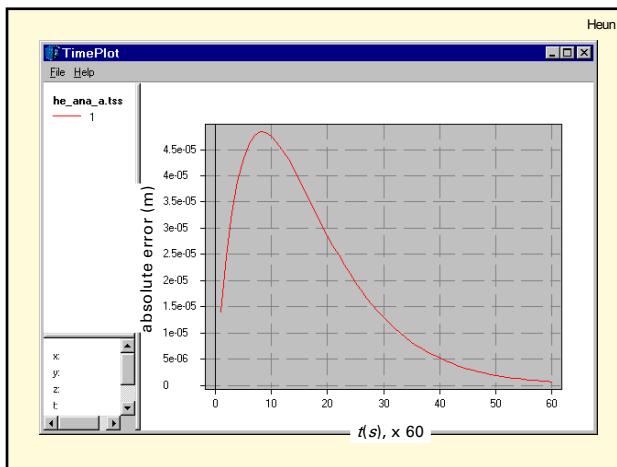
while t < 60
    Ystar = y + Dt * k * y
    y = y + Dt * ((k * y + k * Ystar) / 2)
    print(y)
    t = t + 1

```

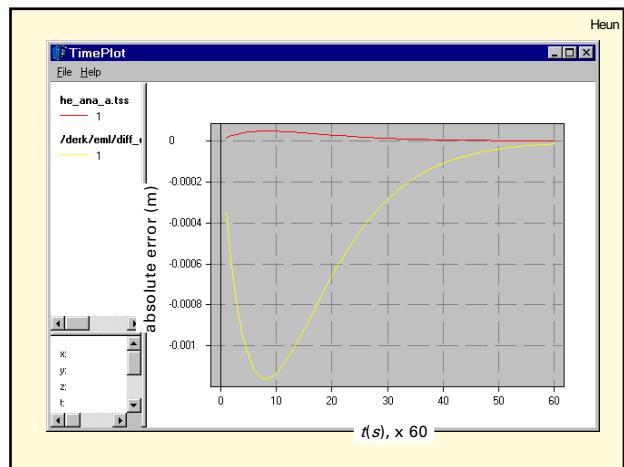
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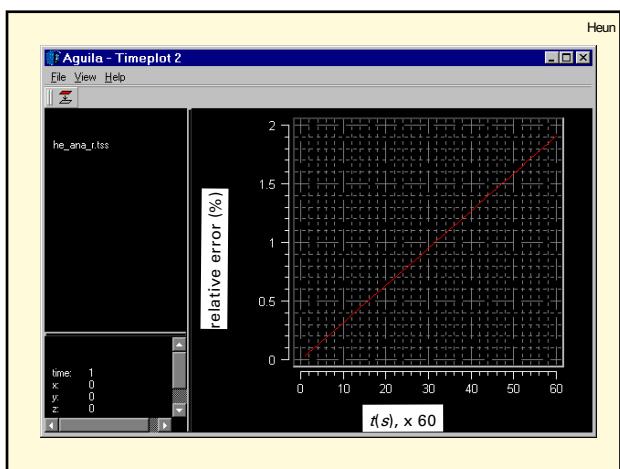
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